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A Methodology for Location-Allocation Problem

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Abstract. In this work we propose a methodology that allows establishing the relationships between the location of the facilities and the clients' allocation with a dense demand. The use of this application lets us know the optimal location of production facilities, warehouses or distribution centers in a geographical space. We solve as well, the customers' dense demand for goods or services; this is, finding the proper location of the facilities in a populated geographic territory, where the population has a demand for services in a constant basis. Finding the location means obtaining the decimal geographical coordinates where the facility should be located, such that the transportation of products or services costs the least. The implications and practical benefits of the results of this work have allowed an enterprise to design an efficient logistics plan in benefit of its supply chain. Firstly, the territory must be partitioned by a heuristic method, due do nature combinatory of the partitioning. After this process, the best partition is selected with the application of factorial experiment design and the surface response methodology. Once the territory has been partitioned into k zones, where the center of each zone is the distribution center, we apply the continuous dense demand function by solving the location-allocation for an area where the population has a dense demand for services.

Keywords. Dense demand, location-allocation, methodology, response surface.

1 Introduction

In a broad sense, it is understood that logistics is found inside the supply chain, and, in general the logistical networks in the supply chain are a system that manage the merchandise network and physical flow among the members of the supply chain influenced by the territorial distribution and by the transportation systems to reduce the logistical expenses and coordinate the production-distribution activities. On this point, the supply chain can be defined as the set of enterprises that comprise providers, manufacturers, distributors and sellers (wholesale or retail) efficiently coordinated by means of collaborative relationships between their key procedures to place the inputs or products requirements in each link of the chain at the right time and at the lowest cost, looking for the biggest impact on the value chains of the members with the end of satisfying the final consumers' requirements. However, in this work we focus on the service location-allocation aspect. This way, the goal of this work is presenting a methodology to support the strategic decision-making process of an enterprise, primarily to locate the facilities for the planning of a logistical network. We have made a special emphasis on the case of georeferenced zones and dense demands, which is a territorial design problem combined with a location-allocation problem.

The methodology is based on finding the proper location of the facilities in a populated geographic territory, where the population continuously demands services. Obtaining the locations means finding the longitude, latitude coordinates of each location point in such a way that the transportation of products and services has a minimum cost.

The problem implies solving the territorial design partition. This partition is obtained with a methodology that begins with the selection of the method of partitioning according to the results generated by the experiment factor and response surface (design of experiments statistical). Then the minimizing the Weber function that has a demand function multiplied by the Euclidean distances as weights is done. The demand function represents the population's demand in every territory, whereas the Euclidean distance is calculated between the potential location points and the demand points.

2 Statistical Design Methodology for Partitioning

The methodology proposed suggests partitioning the territory with a territorial design method that generates compact groups or clusters [10], which has to be obtained first to be able to solve the Location-Allocation Problem (LAP) for a Territorial Design Problem (TDP) with dense demand. The Distribution Centers (DC) allocated will provide services to a group of communities that are found in every geographic area, and each of them is represented by its centroid. The location should be the one that minimizes the travelling expenses by finding the geographical coordinates of the center of the centroids. The populations from these communities represent the potential clients of the DC, and the demand is modeled with continuous demand functions with two variables based on the population density of every group [5].

Due to the numeric nature of the solutions obtained, this problem addresses a continuous case of the LAP. Additionally with the mathematical approach associated, we use a geographical information system (GIS) to create maps of the territories designed [13].

There are many efforts to solve problems related with the location-allocation of services, and the state of the art for it deserves a work of its own [1, 2, 3, 4, 5, 6, 7, 8]. However, we can say that our contribution focuses on presenting a methodology that begins with a territory partitioning process that employs a P-median method and partitioning restrictions to find p-centers by incorporating a metaheuristic [9]. Due to the fact that the population's demand for services is dense, once the distribution centers have been determined, their geographical location (x = longitude, y = latitude) in R^2

must be found such that the services or products transportation has a minimum cost, using the Weber function as stated in the previous section.

For example, let's assume that we wish to know where to locate a healthcare center (assuming as well that all the associated conditions are met). Then for this particular application we could locate clinics in the centroids of every geographic unit and in the center-most point of all the communities, locating a general hospital as DC such that transferring patients requires a minimum amount of time.

First, we have obtained the territory partition, where the cluster formation is based on geometric compactness of territorial design and the minimum distances between centroids [10], i.e, the first part of the methodology consists of the selection of the partition and the heuristic method to continue the statistical experiment (Design of experiments) which will indicate the number of suitable partitions. Design of experiments allows to analyze data using statistical models to observe the interaction between the independent variables and as affect the dependent variable. This methodology is based on experimentation. At the time of these experiments is obtain replicas and randomize data. Using replicas, we have an estimate of experimental error, if higher the number of replicas is, the experimental error is lower. This means that the experiments are given in the same conditions. Randomization during the experiment is essential to avoid the dependence between samples and ensure that results are actually caused by the dependent variables and not by the experimenter.

The first part of the methodology is showed. (Selection of the partition using design of experiments):

1. Select the partition method.
2. Select the candidates of heuristic methods.
   - 2.1. Develop a design of experiments to select the best heuristic method (the one that reach the best cost function).
     - 2.1.1. Estimate effects of the factors (heuristics parameters).
     - 2.1.2. Form an initial model.
     - 2.1.3. Develop testing statistics.
     - 2.1.4. Redesign the model.
     - 2.1.5. Analyze the residuals.
     - 2.1.6. Interpretation of results.
3. Determine the parameters in the selected metaheuristics by identify the number of partitions (groups).
   - 3.1. Select the initial model (Box Bhenken, central composed, etc.).
   - 3.2. Develop experimental tests.
   - 3.3. Analyze the regression model in order exits statistical evidence for the reliability of the experiment.
     - 3.3.1. Verification of the experimental model.
     - 3.3.2. Validation of the parameters.
4. Select the partition to better optimize the cost function.

3 Demand Dense Methodology

Diverse territorial design applications are very useful to solve location problems for services and sales points [11, 12]. For this work, the logistical network design problem with dense demand over a geographical region implies defining (finding) market or services areas, this is a TD application.

The partition of one territory into zones generates *k* zones; this is understood of course as geographical zones grouping. Then, from a logistical point of view, we have available zones to locate facilities that provide services to satisfy the clients' demand. We have chosen a partition of 5 zones for our case study with the goal of allocating Demand Density Functions that we'll denote as DDF, proposed in [6], where DL1,..,DL6 are linear functions and as DNL1 as DNL2 are no linear functions and they are shown in the following table:

Table 1. Demand Density Functions (DDF).

DDF	$D(x, y)$
DL1	$100+7.5x+7.5y$
DL2	$100+10x+5y$
DL3	$100+(100/7)x+(5/7)y$
DL4	$600+(10/3)x+(5/3)y$
DL5	$600+2.5x+2.5y$
DL6	$100+(100/21)x+(5/21)y$
DNL1	$100+(9/80)x^2+(9/80)y^2$
DNL2	$100+(3/1.6\times10^5)x^4+(1.6\times10^5)y^4$

In integral calculus (Stewart 1999) the center of mass of an aluminum sheet that has a density function ρ(x, y) that occupies a region can be obtained (see Fig. 1).

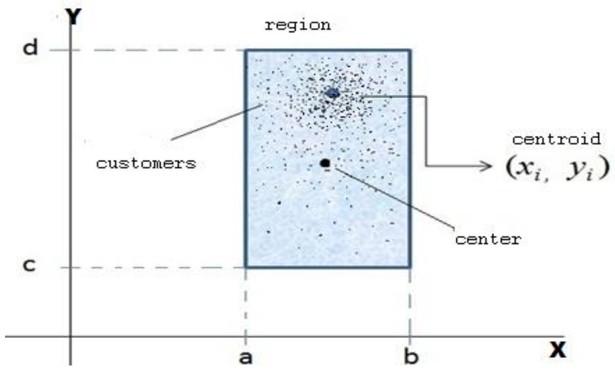

Fig. 1. Center and centroid of a rectangular region in R^2 based on the density of the points.

It's possible to assume that a geographical region in the plane is like a sheet and to obtain its center of mass or centroid through the use of (1), where the density function is a population density function, said point has geographical coordinates for rectangular regions, but in this project we are applying it to irregular regions. The centroids can be interpreted as location points for facilities because the point is located in a place with high population density [1, 2]:

$$x = \frac{\iint x\rho(x,y)dxdy}{\iint \rho(x,y)} \quad ; \; y = \frac{\iint y\rho(x,y)dxdy}{\iint \rho(x,y)}. \tag{1}$$

The formulas to obtain the longitude, latitude coordinates of the location point of every geographical unit, require the use of double integrals over a region R, with density functions with two variables$\rho(x,y)\rho(x,y)$, where R is a geographical region and the density function can be any of the functions from table 1. On the other hand, we need to obtain the integration limits of every geographical unit, by means of a geographical information system (GIS) [1, 8]. The Romberg method has been chosen for double integration. The support software we've employed is free and is known as X numbers, which is an Excel add-on. In this way the coordinates of every centroid for every geographical unit, of territory is obtained.

From a logistical view, each territory that has a population possesses a demand for goods and services and it can defer per zones due to multiple factors. Let's assume that the demand can be modeled by a DDF that associates a demand volume for a certain service to every geographic point. The demand density term is associated to the population density in the zone. Each of the five groups of geographical unit, will be associated with a DDF from table 1 to integrate them to the location model that is described below as a minimization problem.

Let's consider that a centroid of a geographical unit, is a point with geographical coordinates which location depends on the density of population, we can say that $c_j = (x_j, y_j)$ is the representative of the geographical unit.

The solution consists in finding the coordinates $(x,y)(x,y)$ of the point $q \in Gi$ $q \in Gi$ such that the transportation cost from each community to the central facility is minimized.

The mathematical model that represents the conditions mentioned above is written in the following way:

$$\underset{(x,y) \in G_i}{\text{Minimize}} \quad \sum_{j=1}^{|Gi|} |D(x,y)| \sqrt{(x - x_j)^2 + (y - y_j)^2}. \tag{2}$$

The objective function represented in (2) is the total transportation cost TC, known as the Weber function.

Parting from what we have exposed, we express the final model and contribution of our problem:

Given a set of customers distributed within a territory $T \subseteq R^2$ and $P = \{G_1, G_2, \ldots, G_k, \ldots G_p\}$ a partition of T into p clusters, each G_k is a cluster of Agebs for $k = 1,2,\ldots,p$. Each geographical object has a representative called centroid $c_j = (x_j, y_j,)$ from which each community is served. Each point $q = (x,y) \in T$, has a density of demand given by $D(q) = D(x,y)$. Let $d(q,c_j)$, be the Euclidean distance from any point to the centroid. The cost of transportation from a point q to the centroid c_j is defined as $D(q)d(q,c_j)$.

The solution consists in finding the coordinates from a point $(x,y) \in G_i$ such that the transportation cost is minimized from each community to a central facility (like equation 2). The mathematical model that represents the mentioned conditions is the following:

$$\underset{(x,y)\in G_i}{\text{Minimize}} \quad Z = \sum_{j=1}^{Gi} |D(x,y)| \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2}, \tag{3}$$

Subject to

$$\bigcup_{i=1}^{p} G_i = T, \tag{4}$$

and

$$\bigcup_{j=1}^{|G_i|} A_j = G_i \quad \forall\ i = 1, 2, \dots, p. \tag{5}$$

The objective function Z represented in (3) is the total transportation cost and (4) and (5), are the constraints of the partitioning of the territory T and the sub territories G_i. The sequence of necessary steps to obtain the coordinates of the central facility in a cluster is as follows:

1. Define the parameters to partition the territory T.
2. Generate the partition with the VNS metaheuristic.
3. With the file obtained generate a map inside a map with a GIS.
4. Associate to the chosen cluster $P_i, i = 1,2,\dots,p$ $P_i, i = 1,2,\dots,p$, a demand density function $D(x,y)$. $D(x,y)$.
5. Calculate the centroids of each AGEB, using (4).
6. Apply (3) for the chosen cluster.

$$x_i = \frac{\iint x\rho(x,y)dxdy}{\iint \rho(x,y)dxdy} \quad \text{and} \quad y_i = \frac{\iint y\rho(x,y)dxdy}{\iint \rho(x,y)\,dxdy}\,. \tag{6}$$

Equations (6) were rewritten to have an order in the methodology. This equation is also (1) and the equations above are the classical formulas of calculus used to calculate the centroid of a metallic plate with density ρ, in this paper we take the Agebs as metallic plates and a population density given by $\rho(x,y)$

4 Results

We have applied this methodology to one territory and we can establish that the solutions obtained for the LAP model for TDP with dense demand are consistent with the geographic location of the region. We obtained the coordinates of eight possible location points, depending on the demand density function, which associates itself to the geographical units belonging to the cluster under study selected from the partition of the territory. Fig. 3 shows the location of nine points. Therefore, we can select any point as the location point of the cluster; we can also state that the demand density function does not influence the location of the centers of the centroids. The ninth point which coordinates are $P_c = (x_{cc}, y_{cc})$ this point is calculated as the point which distance to any center is the minimum we can consider as the center of centers, and as the best point of location of the whole cluster of geographical units, without losing generalities. In a more general way, the location point can be any point within the circle $(x - x_{cc})^2 + (y - y_{cc})^2 = r^2$, where $r = max\{ d(P_c, c_j) \mid j = 1, 2, 3, \dots, n\}$. The proposal in this article gives a structure to solve location allocation models based on geographic information systems as it's reflected in the any case study.

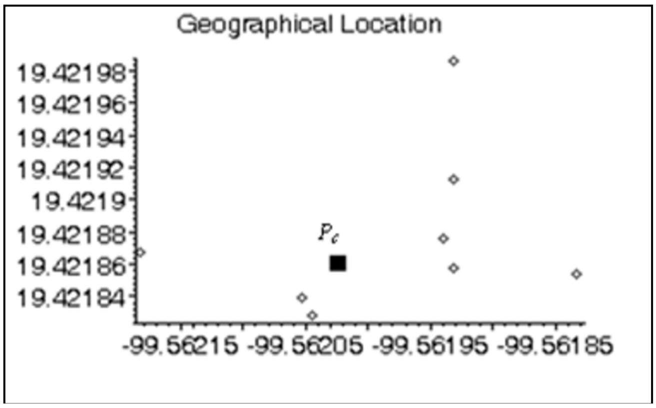

Fig. 2. Location that minimizes the objective function.

The methodology was tested in demand regions of irregular shape in comparison with previous papers where the regions are rectangular or convex polygons. According to the analysis of the results obtained, the inclusion of the territorial design aspects with the use of density functions in location-allocation models, gives a greater range of possible applications to real problems, for example in the design of a supply chain, among others. The integration of diverse tools such as metaheuristics, geographic information systems and mathematical models provide a strong methodology in visual environments such as maps. Another contribution of this paper is the consideration of three relevant aspects: territorial design, location-allocation, and dense demand.

In general, the proposal presented contributes to the decision-making process in logistical problems when the population's demand is implicit. As a case study we chose Metropolitan Zone, however an advantage of our methodology consists in that it can handle other kinds of geographical data such as blocks, districts or states.

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